

All Saints' School Calculation Policy

The following calculation policy is based around the requirements and expectations of the National Curriculum 2014.

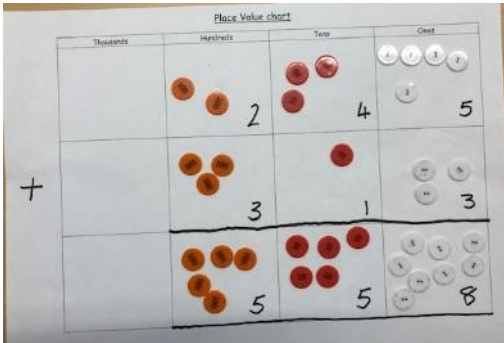
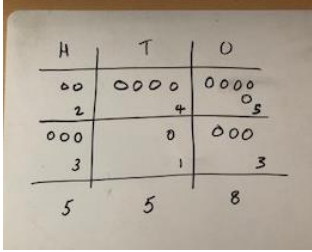
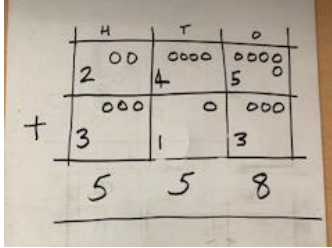
Its aim is to provide a smooth progression between year groups and teacher helping children to build on the skills that they develop during their time at All Saints'.

The National Curriculum for mathematics aims to ensure that all pupils:

- become fluent in the fundamentals of mathematics, including through varied and frequent practice with increasingly complex problems over time, so that pupils develop conceptual understanding and the ability to recall and apply knowledge rapidly and accurately
- reason mathematically by following a line of enquiry, conjecturing relationships and generalisations, and developing an argument, justification or proof using mathematical language
- can solve problems by applying their mathematics to a variety of routine and non-routine problems with increasing sophistication, including breaking down problems into a series of simpler steps and persevering in seeking solutions

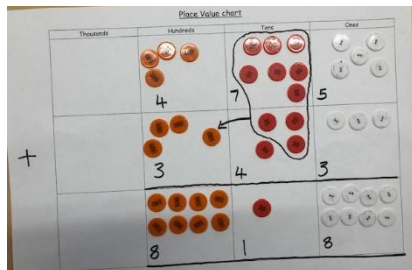
The National Curriculum 2014 requires all children to master the skills that they are developing before moving onto the next stage of their learning. So whilst the following information will be set out by year group it is important to remember that all children will need to be taught relative to their current level of attainment before moving on to the next objective.

Addition - Year 3

Objective	Concrete	Pictorial	Abstract									
<p>add with up to three digits, using formal written methods of columnar addition</p> <p>Without Regrouping</p>	<p>Using manipulatives such as place value counters, Cuisenaire rods, place value blocks, Numicon children line up hundreds, tens and ones.</p> <div style="text-align: center;"> <table border="1" style="margin: auto;"> <thead> <tr> <th>H</th> <th>T</th> <th>O</th> </tr> </thead> <tbody> <tr> <td style="text-align: center;">●●</td> <td style="text-align: center;">●●●●</td> <td style="text-align: center;">●●●●●</td> </tr> <tr> <td style="text-align: center;">●●●●</td> <td style="text-align: center;">●</td> <td style="text-align: center;">●●●</td> </tr> </tbody> </table> </div> <p style="text-align: center;">$245 + 313 = 558$</p> <p>Children are encouraged to write the calculation alongside the representation</p> 	H	T	O	●●	●●●●	●●●●●	●●●●	●	●●●	<p>Children are encouraged to write down the calculation similar to how it has been modelled</p>  <p>They can then move on to using a layout similar to columnar addition</p> 	<p>They move on to just using the formal method, they may need to use an expanded step before being completely secure with their understanding of the concrete method.</p> $ \begin{array}{r} 200 + 40 + 5 \\ + 300 + 10 + 3 \\ \hline 500 + 50 + 8 \\ \hline \end{array} $ $ \begin{array}{r} 245 \\ + 313 \\ \hline 558 \end{array} $
H	T	O										
●●	●●●●	●●●●●										
●●●●	●	●●●										

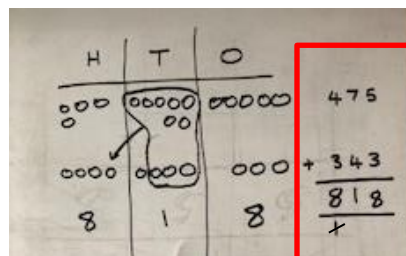
With Regrouping

$$475 + 343 = 818$$



Once the counters are combined, the children can exchange 10 tens counters for a hundred counter giving them the information they need to complete the addition calculation. Being able to see this in front of them helps in their understanding.

Children are given the opportunity to model the calculation without counters circling the counters and then showing where they can be exchanged



This can then be extended to modelling the written method alongside the calculation

Children may require an expanded step before moving onto the concrete method

$$\begin{array}{r} 400 + 70 + 5 \\ + 300 + 40 + 3 \\ \hline 700 + 110 + 8 \end{array}$$

Children complete the formal method with exchanging where necessary.

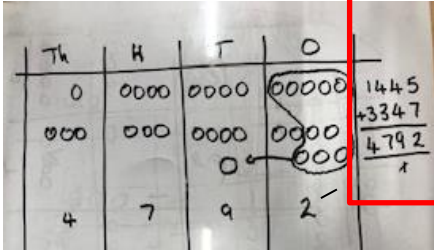
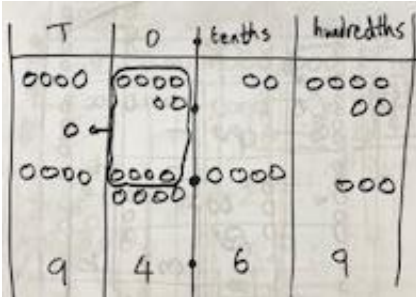
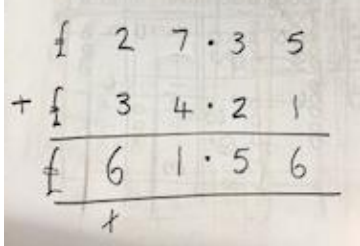
$$\begin{array}{r} 475 \\ + 343 \\ \hline 818 \\ \hline 1 \end{array}$$

Calculations can be provided to the children where exchanging can happen in one or all of the columns. Usually children progress from exchanging with the one to tens to hundreds and then multiple columns.

VOCABULARY

addition add, more, and make, sum, total, altogether, double, near double, half, halve, one more, two more ... ten more ... one hundred more, how many more to make ...? how many more is ... than ...? how much more is ...? equals, is the same as, number bonds/pairs/facts, missing number, tens boundary, hundreds boundary, addend

Addition - Year 4

Objective	Concrete	Pictorial	Abstract												
<p>add numbers with up to 4 digits using the formal written method of columnar addition</p>	<p>Using manipulatives such as place value counters, Cuisenaire rods, place value blocks, Numicon children line up hundreds, tens and ones.</p> <table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 25%;">Th</th> <th style="width: 25%;">H</th> <th style="width: 25%;">T</th> <th style="width: 25%;">O</th> </tr> </thead> <tbody> <tr> <td>●</td> <td>●●●●</td> <td>●●</td> <td>●●●●</td> </tr> <tr> <td>●●●●</td> <td>●●●●</td> <td>●●●●</td> <td>●●●●</td> </tr> </tbody> </table> <p style="text-align: center;">$1445 + 3347 = 4792$</p> <p>Children are encouraged to write the calculation alongside the representation</p>	Th	H	T	O	●	●●●●	●●	●●●●	●●●●	●●●●	●●●●	●●●●	<p>Children are given the opportunity to model the calculation without counters circling the counters and then showing where they can be exchanged to</p>  <p>This can then be extended to modelling the written method alongside the calculation</p>	<p>Children complete the formal method with exchanging where necessary.</p> $\begin{array}{r} 1445 \\ + 3347 \\ \hline 4792 \\ \times \end{array}$
Th	H	T	O												
●	●●●●	●●	●●●●												
●●●●	●●●●	●●●●	●●●●												
<p>including decimals</p>	<table border="1" style="margin: 10px auto; border-collapse: collapse; text-align: center;"> <thead> <tr> <th style="width: 25%;">T</th> <th style="width: 25%;">O</th> <th style="width: 25%;">0.1</th> <th style="width: 25%;">0.01</th> </tr> </thead> <tbody> <tr> <td>●●</td> <td>●●●●</td> <td>●●</td> <td>●●●●</td> </tr> <tr> <td>●●●●</td> <td>●●●●</td> <td>●●</td> <td>●●●●</td> </tr> </tbody> </table> <p style="text-align: center;">$46.26 + 48.43 = 94.69$</p> <p>This can also be related to money.</p>	T	O	0.1	0.01	●●	●●●●	●●	●●●●	●●●●	●●●●	●●	●●●●	 <p>Again children should be encouraged to model the written calculation alongside their working.</p>	
T	O	0.1	0.01												
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●●●●	●●●●	●●	●●●●												

VOCABULARY

addition add, more, and make, sum, total, altogether, double, near double half, halve one more, two more... ten more... one hundred more how many more to make ...? how many more is ... than ...? how much more is ...? equals, is the same as, number bonds/pairs/facts, missing number, ones boundary, tens boundary, hundreds boundary, thousands boundary, tenths boundary, addend


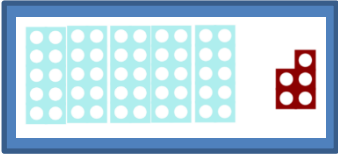
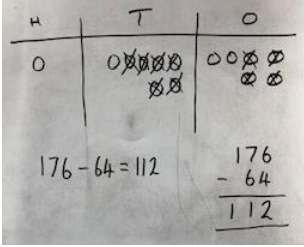
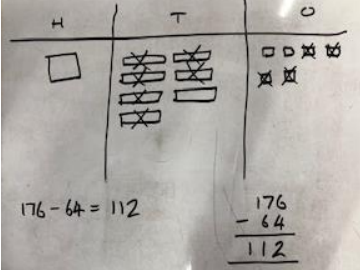
Addition - Year 5/6

Objective	Concrete	Pictorial	Abstract
add whole numbers with more than 4 digits using formal written methods	See Year 4 and extend columns where necessary	See Year 4 and extend columns where necessary	$ \begin{array}{r} 144\ 562 \\ + 502\ 489 \\ \hline 547\ 051 \\ \hline \cancel{1}\ \cancel{1} \end{array} $
add several numbers of increasing complexity, including adding money, measure and decimals with different numbers of decimal points	See Year 4 and extend rows if necessary	See Yr 4 and extend rows here necessary	<p>Children can apply the formal method to suit the needs of the calculation</p> $ \begin{array}{r} \pounds\ 14.45 \\ \pounds\ 3.47 \\ + \pounds\ 524.89 \\ \hline \pounds\ 542.82 \\ \hline \cancel{1}\ \cancel{1}\ \cancel{2} \end{array} $ $ \begin{array}{r} 218.452 \\ 73.820 \\ + 1524.800 \\ \hline 1817.072 \\ \hline \cancel{1}\ \cancel{2} \end{array} $ <p>Children can insert place holders to aid with the lay out of the calculation</p>

VOCABULARY

addition add, more, and make, sum, total, altogether, double, near double half, halve one more, two more... ten more... one hundred more how many more to make ...? how many more is ... than ...? how much more is ...? equals, is the same as, number bonds/pairs/facts, missing number, ones boundary, tens boundary, hundreds boundary, thousands boundary, etc ... tenths boundary, hundredths boundary, addend

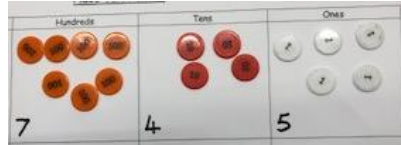
Subtraction - Year 3

Objective	Concrete	Pictorial	Abstract
<p>To subtract numbers with up to three-digits, using formal written methods of columnar subtraction</p> <p>Column subtraction (without exchanging)</p>	<p>Use base 10 or Numicon to model the subtraction</p> <p>55 - 22</p>  	<p>Once children are secure with Base 10 / Numicon and PV counters they can move on to recording pictorially</p>  <p>Children are encouraged to write the calculation down alongside their representation</p> <p>They could also draw base ten representations if they wanted to</p> 	<p>Children may need to use an expanded step before being completely secure with their understanding of the concrete method.</p> <p>58 - 26 = 32</p> $\begin{array}{r} 50 + 8 \\ - 20 + 6 \\ \hline 30 + 2 \\ \hline \end{array}$ $\begin{array}{r} 58 \\ - 26 \\ \hline 32 \end{array}$

Column subtraction
(with exchanging / regrouping)

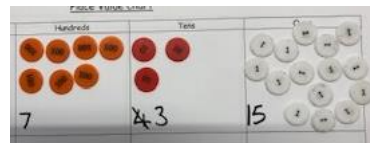
Model the process of $745 - 317$ using base 10 or PV counters. Ensure the language of exchanging and regrouping is used.

Step 1:



Start with the 1's column. Can 7 be taken from 5 easily? No - then exchange a ten for ten ones and regroup them into the correct column.

Step 2:



Once this step is completed children can subtract / take off the relevant counters from each column.

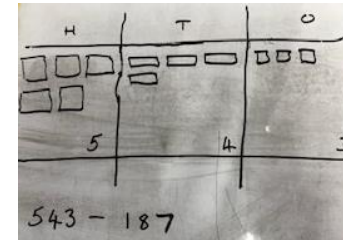
Carefully chosen questions can extend children to exchanging and regrouping next in the tens column, then in both the tens and ones columns as appropriate.

They could also decompose a number

Children to model using their whiteboards:

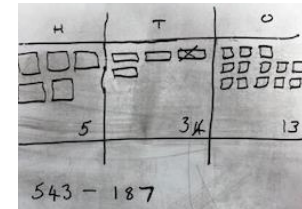
Step 1:

Set up calculation



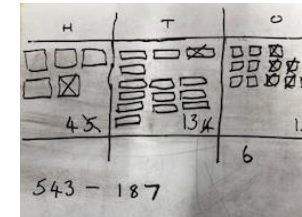
Step 2:

Exchange and regroup 10's for 1's



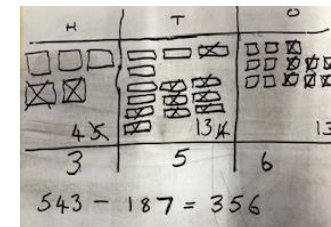
Step 3:

Cross out ones to indicate subtraction has taken place, write new amount of ones underneath and then focus on tens column exchanging and regrouping a hundred for ten tens

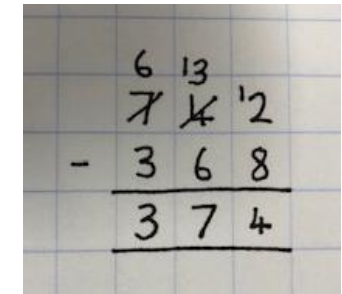
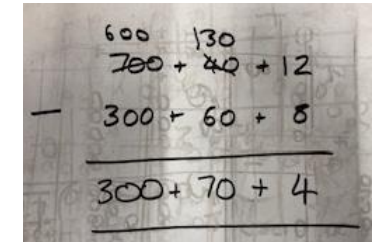


Step 4:

Cross out the tens to show subtraction, write the amount left underneath and then repeat with the hundreds column to complete the calculation.



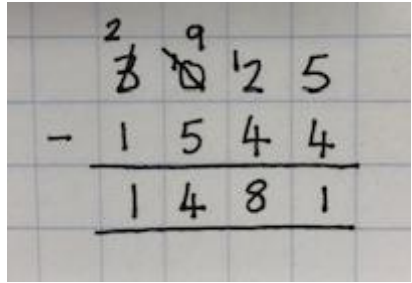
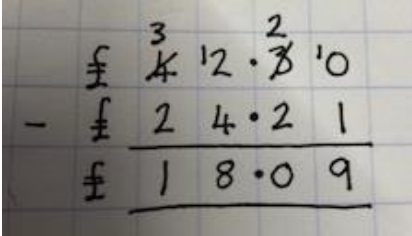
Children may need to use an expanded step before being completely secure with their understanding of the concrete method.



VOCABULARY

minuend, subtrahend, equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units, exchange, decomposition, regroup, how many have gone? one less, two less, ten less ... one hundred less how many fewer is ... than ...?

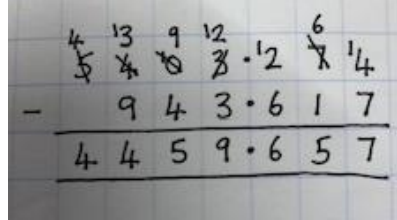
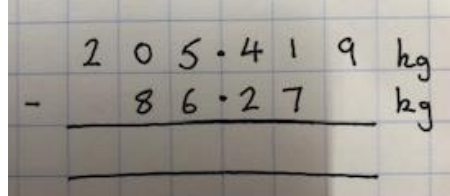
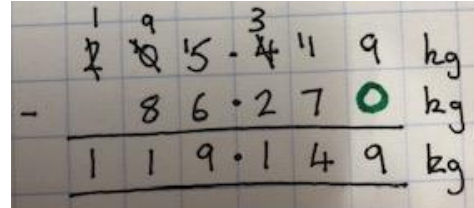
Subtraction - Year 4

Objective	Concrete	Pictorial	Abstract
Subtract numbers with up to 4 digits using the formal written methods appropriate of columnar subtraction where appropriate Year 4 subtraction with up to 4 digits.	See Year 3 with extension into the thousand's column	See Year 3 with extensions into the thousand's columns	
Introduce decimal subtraction through context of money	See Year 3 with extension into the tenths and hundredths columns	See Year 3 with extensions into the tenths and hundredths columns	

VOCABULARY

minuend, subtrahend, equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units, exchange, decomposition, regroup, how many have gone? one less, two less, ten less ... one hundred less how many fewer is ... than ...? Inverse, ones boundary, tens boundary, hundreds boundary, tenths boundary

Subtraction - Year 5/6

Objective	Concrete	Pictorial	Abstract
<p>Subtract with at least 4 digits, including money and measures.</p> <p>Subtract with increasingly large and more complex numbers and decimal values (up to 3 decimal place).</p>	<p>See Year 3 with extensions into the relevant columns where necessary</p>	<p>See Year 3 with extensions into the relevant columns where necessary</p>	<div style="text-align: center;">  </div> <div style="text-align: center; margin-top: 20px;">  </div> <p style="text-align: center; margin-top: 10px;">When the amount to subtract is different in size then children can put in a 0 to help complete the calculation</p> <div style="text-align: center;">  </div>

VOCABULARY

minuend, subtrahend, equal to, take, take-away, less, minus, subtract, leaves, distance between, how many more, how many fewer/less than, most, least count back, how many left, how much less is...difference, count on, strategy, partition, tens units, exchange, decomposition, regroup, how many have gone? one less, two less, ten less ... one hundred less how many fewer is ... than ...? Inverse, ones boundary, tens boundary, hundreds boundary etc ... tenths boundary, hundredths boundary

Multiplication - Year 3

Objective

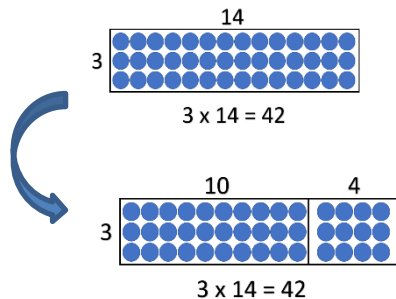
Multiplying two digit number by a one digit number

Grid method progressing to the formal method.

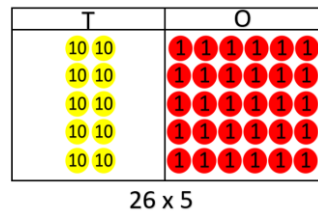
Concrete

Children to make the links between arrays and multiplication building on their knowledge from last year

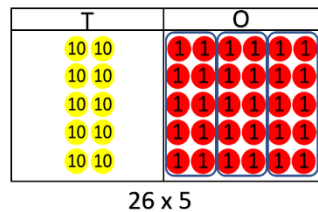
3x14 can be arranged in a grid to make calculating it easier



Children can use place value counters, base 10 or any other resource to calculate groups of a number



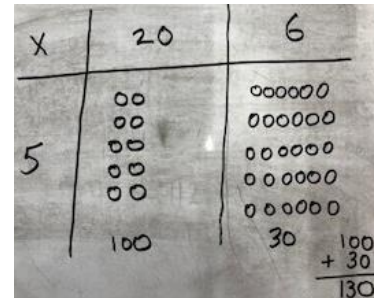
They can then make any exchanges that they need to



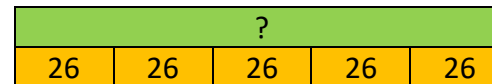
Pictorial

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below



Bar Models can also be used as an appropriate representation which may help to cement the relationship with repeated addition and also support learners when solving problems.



Abstract

Children can use the grid method to calculate e.g.

$$3 \times 24$$

x	20	4
3	60	12

$$60 + 12 = 72$$

Before moving on to the expanded method:

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 12 \quad (4 \times 3) \\ 60 \quad (20 \times 3) \\ \hline 72 \end{array}$$

then the compact method:

$$\begin{array}{r} 24 \\ \times 3 \\ \hline 72 \\ \hline \end{array}$$

T		O
10	10	
10	10	10
10	10	10
10	10	10
10	10	

$$26 \times 5$$

H	T	O
100		
		10
		10
		10

$$26 \times 5 = 130$$

Giving them the final answer

When introducing any new calculation method, it is always important to start with questions that involve no regrouping before introducing it slowly and only with one column at a time. This will help to build the children's confidence in the method leading to a more secure understanding.

VOCABULARY

multiplication, multiply, multiplied by, multiple, factor, groups of, times, multiplier, multiplicand, product, once, twice, three times ... ten times repeated addition, array, row, column, number patterns, multiplication table, multiplication fact,

Multiplication - Year 4

Objective

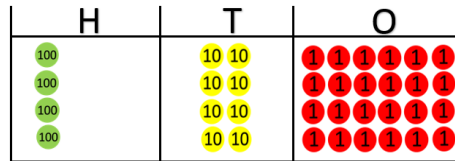
Grid method recap from Year 3 for 2 digits x 1 digit moving on to 3 digit x 1 digit number

Multiplying 3-digit numbers by 1 digit

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

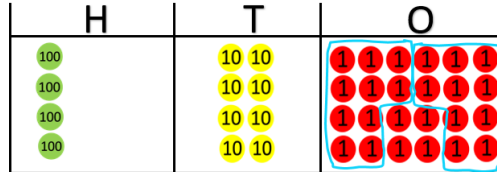
Concrete

Place value counters can be used to explore the calculation. As the calculation is 126×4 then there needs to be 4 rows. The number is partitioned and the rows are created with the place value counters

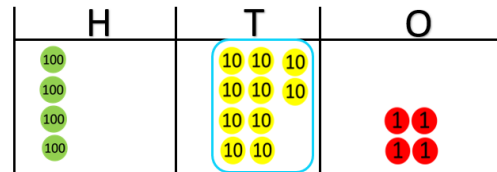


$$4 \times 126$$

Any exchanges are then made before adding up the rows.

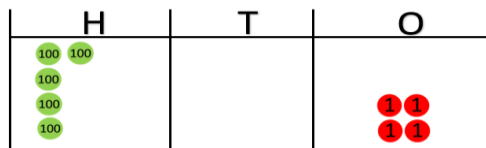


$$4 \times 126$$



$$4 \times 126$$

Giving the final answer



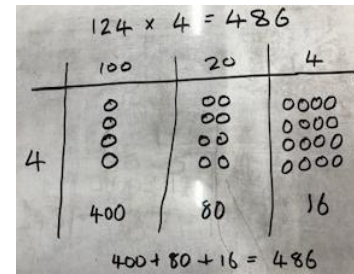
$$4 \times 126 = 504$$

Pictorial

Children can represent their work with place value counters in a way that they understand.

They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below

(as an extension of what they learnt in Yr 3)



The grid method can be used to support the expanded form and help children to recognise the links between the two

Abstract

Children can use the grid method to calculate e.g.

$$4 \times 124$$

x	100	20	4
4	400	80	16

$$400 + 80 + 16 = 496$$

(This could be added using the column method although quick mental additions would be more efficient)

Before moving on to a more formal method:

$$\begin{array}{r}
 124 \\
 \times 4 \\
 \hline
 16 \quad (4 \times 4) \\
 80 \quad (20 \times 4) \\
 400 \quad (100 \times 4) \\
 \hline
 496
 \end{array}$$

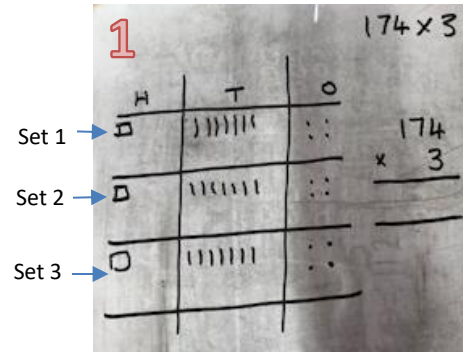
then the compact method:

$$\begin{array}{r}
 124 \\
 \times 4 \\
 \hline
 496 \\
 \hline
 \cancel{1}
 \end{array}$$

The concrete and pictorial methods can also be used to further support the compact method. It is important to remind children to calculate first with the ones column and then move up to help support understanding of the compact method.

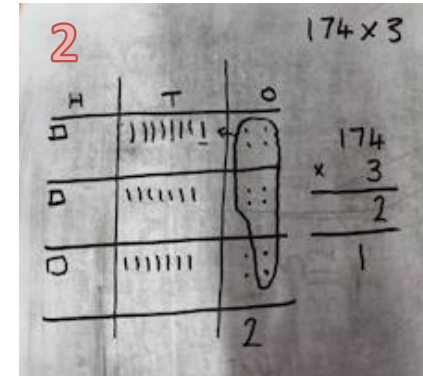
Step 1:

Set out the calculations alongside each other
There are 3 sets of 174



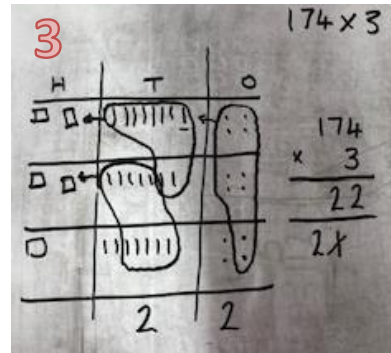
Step 2:

Make the exchange in the ones column (if needed) and show the link between that and the multiplication calculation in the compact method, recording as necessary



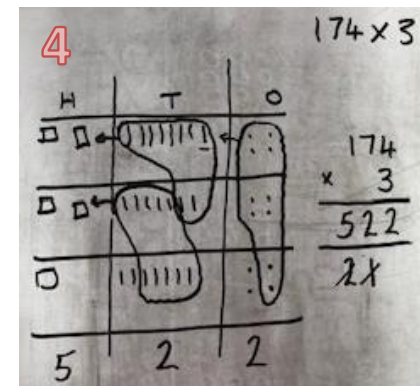
Step 3:

Move on to the tens column and repeat with the exchanges where needed and show the link between that and the multiplication calculation in the compact method recording as necessary



Step 4:

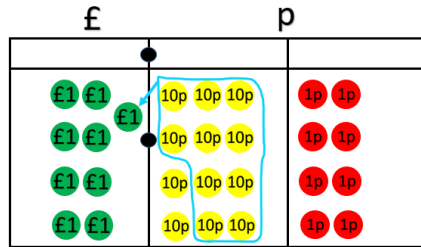
Complete the hundreds column again making the link between the pictorial and the Calculation before recording



Multiplication involving decimals (inc. money)

Multiply decimal up to 2 decimal places by a single digit.

Use manipulatives to calculate the amount.
 Make exchanges where necessary.
 Total up the final amount.
 Again, use the written example alongside the manipulative wherever possible.

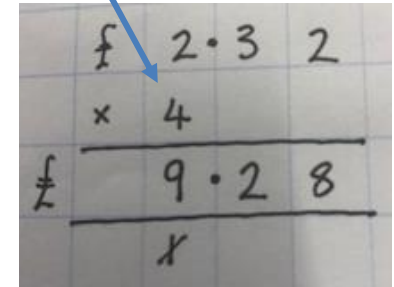


$$£2.32 \times 4 = £9.28$$

The grid method could be used to provide extra support however teachers and children must be wary of how the children record the amounts, or convert amounts, to ensure mistakes are not made.

x	£2	30p	2p
4	£8	120p or £1.20	8p

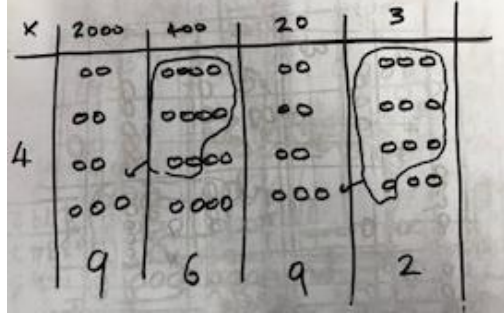
Teachers to ensure the correct placement of the 4 in terms of a decimal calculation



VOCABULARY

multiplication, multiply, multiplied by, multiple, factor, groups of, times, multiplier, multiplicand, product, once, twice, three times ... ten times repeated addition, array, row, column, number patterns, multiplication table, multiplication fact, square, squared, cube, cubed

Multiplication - Year 5/6

Objective	Concrete	Pictorial	Abstract										
<p>Column multiplication for 3 and 4 digits x 1 digit</p> <p>Multiply numbers up to 4-digits by a one-digit number using the format written method, (including long multiplication for 2-digit numbers – see below)</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>It is important to remind children to calculate first with the ones column and then move up to help support understanding of the compact method.</p> <p>(see Yr 4)</p>	<p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below (as an extension of what they learnt in Yr 4)</p> 	<p>Children can use the grid method to calculate e.g.</p> <table border="1" data-bbox="1657 347 2060 430"> <tr> <td>x</td> <td>2000</td> <td>400</td> <td>20</td> <td>3</td> </tr> <tr> <td>4</td> <td>8000</td> <td>1600</td> <td>80</td> <td>12</td> </tr> </table> <p>$8000 + 1600 + 80 + 12 = 9692$ (This could be added using the column method or mentally)</p> <p>Before moving on to a more formal method:</p> $ \begin{array}{r} 2423 \\ \times \quad 4 \\ \hline 12 \quad (3 \times 4) \\ 80 \quad (20 \times 4) \\ 1600 \quad (400 \times 4) \\ 4000 \quad (2000 \times 4) \\ \hline 9692 \end{array} $ <p>then the compact method:</p> $ \begin{array}{r} 2423 \\ \times \quad 4 \\ \hline 9692 \\ \cancel{1} \quad \cancel{1} \end{array} $	x	2000	400	20	3	4	8000	1600	80	12
x	2000	400	20	3									
4	8000	1600	80	12									

Column multiplication
(long multiplication)

Manipulatives can still be used to support calculations – but should always be used alongside the multiplication model

The grid method can be used to support the understanding of the rows alongside the method for long multiplication

Here the rows from the grid method are shown using the brackets. Eventually these will not be needed.

$$23 \times 17 = 391$$

x	20	3	
10	200	30	230
7	140	21	+ 161
			391

$$23 \times 17 = 391$$

x	17	
1, 6, 1	(23 × 7)	
230	(23 × 10)	
391		

$$349 \times 23 = 8027$$

x	300	40	9	
20	6000	800	180	6980
3	900	120	27	+ 1047
				8027

$$349 \times 23 = 8027$$

x	23	
1047	(349 × 3)	
6980	(349 × 20)	
8027		

Children will also complete calculations involving decimals. See how multiplier obeys the value of the column it is in, in relation to the multiplicand.


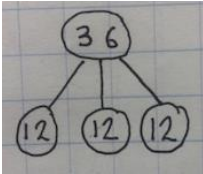
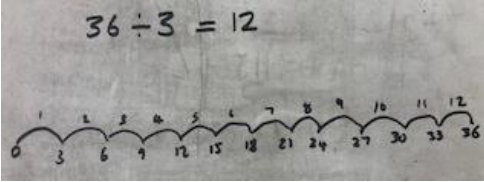
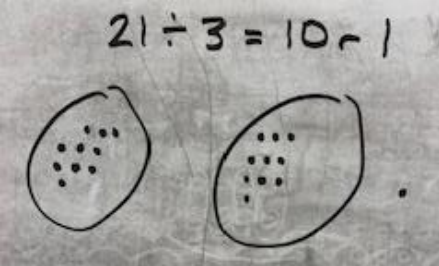
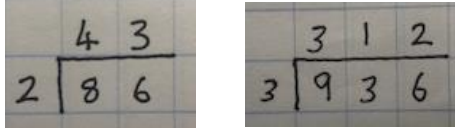
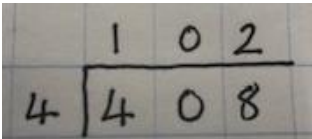
$$4.739 \text{ kg} \times 23 = 108.997$$

x	23	
1, 4, 2, 1, 7		
9, 4, 7, 80		
108.997	kg	

VOCABULARY

multiplication, multiply, multiplied by, multiple, factor, groups of, times, multiplier, multiplicand, product, once, twice, three times ... ten times repeated addition, array, row, column, number patterns, multiplication table, multiplication fact, square, squared, cube, cubed

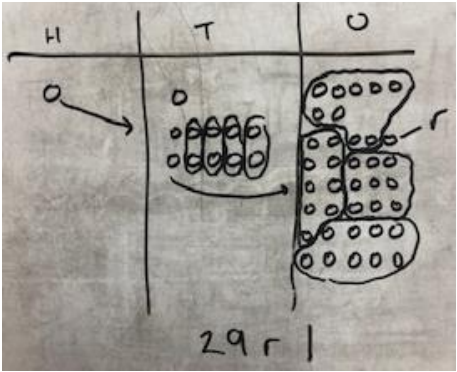
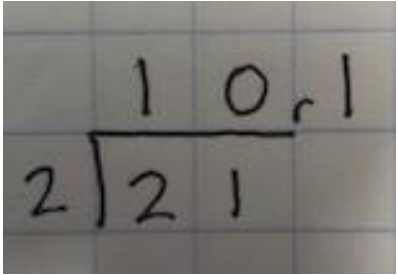
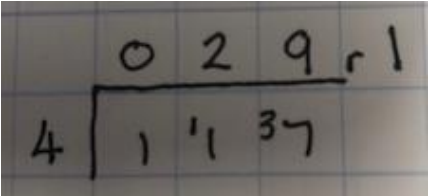
Division - Year 3

Objective	Concrete	Pictorial	Abstract
<p>division using known multiplication facts</p>	<p>Children can use Numicon/ Base 10/Place value Counters to identify division facts</p> <p>$24 \div 4 = 6$</p>  <p>Numicon can be used to support known multiplication facts if children are struggling.</p> <p>Children can use place value counters and then split the number up into 3 groups starting with the 10s. They can exchange if necessary and then split up the remaining ones. This can be a big help when moving onto the short division method</p>	<p>$36 \div 3 = 12$</p> <p>Children can use the part – whole model</p>  <p>Or draw a number line to help solve the calculation</p>  <p>Children can use the dots in circles method as well to support but should be encouraged to count in multiples and move on to a more efficient method</p> 	<p>Begin with divisions that divide equally with no remainder</p>  <p>Children should be aware that a 0 is used to keep place value, if the number is not divisible.</p> 

VOCABULARY

dividend, divisor, quotient, division, dividing, divide, divided by, divided into, left, left over, remainder, grouping, sharing, share, share equally, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, doubling, halving, array row, column number patterns, division fact, inverse,

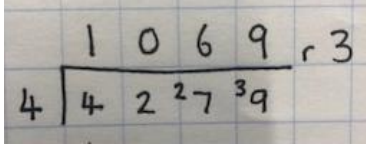
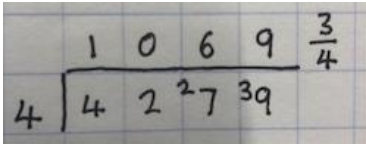
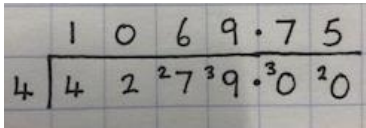
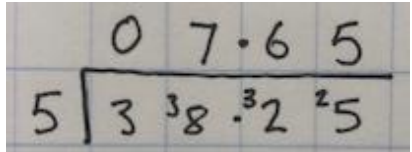
Division - Year 4

Objective	Concrete	Pictorial	Abstract																											
<p>Divide up to 3 digit numbers by 1 digit. Short Division</p> <p>inc. division with remainders</p>	<p>Children can use PV Counters/base 10 or Numicon to help calculate.</p> <p>Exchanging may be needed to support calculations</p> <p>Step 1 Create amount</p> <div style="text-align: center;"> $117 \div 4 =$ <table border="1" style="margin: auto;"> <tr><td style="width: 33%;">H</td><td style="width: 33%;">T</td><td style="width: 33%;">O</td></tr> <tr><td style="text-align: center;">100</td><td style="text-align: center;">10</td><td style="text-align: center;">1111 1111</td></tr> </table> </div> <p>Step 2 Exchange 100's</p> <div style="text-align: center;"> $117 \div 4 =$ <table border="1" style="margin: auto;"> <tr><td style="width: 33%;">H</td><td style="width: 33%;">T</td><td style="width: 33%;">O</td></tr> <tr><td style="text-align: center;">100</td><td style="text-align: center;">10 10 10 10 10 10 10 10 10 10 10</td><td style="text-align: center;">1111 1111</td></tr> </table> </div> <p>Step 3 Group 10s and exchange the rest</p> <div style="text-align: center;"> $117 \div 4 =$ <table border="1" style="margin: auto;"> <tr><td style="width: 33%;">H</td><td style="width: 33%;">T</td><td style="width: 33%;">O</td></tr> <tr><td></td><td style="text-align: center;">10 10 10</td><td style="text-align: center;">1111 1111 1111 1111 1111 1111 1111</td></tr> <tr><td style="text-align: center;">10 10</td><td style="text-align: center;">10 10</td><td style="text-align: center;">10 10</td></tr> </table> </div> <p>Step 4 Group 1's identify the remainder Complete calculation</p> <div style="text-align: center;"> $117 \div 4 = 29 \text{ r } 1$ <table border="1" style="margin: auto;"> <tr><td style="width: 33%;">H</td><td style="width: 33%;">T</td><td style="width: 33%;">O</td></tr> <tr><td></td><td></td><td style="text-align: center;">1</td></tr> </table> </div>	H	T	O	100	10	1111 1111	H	T	O	100	10 10 10 10 10 10 10 10 10 10 10	1111 1111	H	T	O		10 10 10	1111 1111 1111 1111 1111 1111 1111	10 10	10 10	10 10	H	T	O			1		<p>Children can start with remainders that are simple and can be seen in other ways to help make links between calculations they are familiar with and the short division method</p> <div style="text-align: center;">  </div> <div style="text-align: center; margin-top: 20px;">  </div>
H	T	O																												
100	10	1111 1111																												
H	T	O																												
100	10 10 10 10 10 10 10 10 10 10 10	1111 1111																												
H	T	O																												
	10 10 10	1111 1111 1111 1111 1111 1111 1111																												
10 10	10 10	10 10																												
H	T	O																												
		1																												
		<div style="border: 2px solid blue; border-radius: 50%; padding: 10px; width: fit-content; margin: auto;"> <p>Children can become familiar with exchanging and carrying over any amounts that can not be divided by linking in how the numbers are exchanged to the amount recorded in the new column. It is important that whenever the children are using manipulatives that they are encouraged to write the equivalent calculation alongside to help develop these links and understanding</p> </div>																												

VOCABULARY

dividend, divisor, quotient, division, dividing, divide, divided by, divided into, left, left over, remainder, grouping, sharing, share, share equally, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, doubling, halving, array row, column number patterns, division fact, inverse

Division - Year 5/6

Objective	Concrete	Pictorial	Abstract
<p>divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders</p>	<p>As Year 4</p>	<p>As Year 4</p>	<p>Children understand that remainders can be interpreted in a number of ways depending on the context.</p> <p>They can either be a straight remainder</p>  <p>Presented as a fraction</p>  <p>Or completed as a decimal by extending into decimal places</p>  <p>Children should also be made aware of how decimal numbers can also be divided using this method</p> 

divide numbers up to 4 digits by a two-digit whole number using the formal written method of **long division**, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

By the time children learn this method they should be secure in their understanding of short division and how to complete calculations quickly and effectively.

When starting to approach Long division starting with a single digit as the divisor can help children become familiar with the method. Taking out the calculation worries can help to make the children more comfortable

<p>1. DIVIDE</p> $\begin{array}{r} \text{T O} \\ 2 \overline{) 58} \\ \underline{4} \\ 18 \end{array}$ <p>2 goes into 5 2 times or 5 tens $\div 2 = 2$ tens But there is a remainder!</p>	<p>2. MULTIPLY AND SUBTRACT</p> $\begin{array}{r} \text{T O} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>To find the remainder multiply 2 by 2 and get 4. Write that underneath the tens column and subtract it to get the remainder of 1 ten</p>	<p>3. DROP</p> $\begin{array}{r} \text{T O} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Next drop down the 8 in the ones column and write it next to the 1 in the tens. Combine them and you have 18</p>
<p>1. DIVIDE</p> $\begin{array}{r} \text{T O} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Divide 2 into 18, place 9 in the quotient</p>	<p>2. MULTIPLY AND SUBTRACT</p> $\begin{array}{r} \text{T O} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$ Write that 18 underneath and subtract to get 0</p>	<p>3. DROP</p> $\begin{array}{r} \text{T O} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>
<p>The same method can then be transferred to two digit dividends. Using this link will help children to become more secure with long division.</p>		

VOCABULARY

dividend, divisor, quotient, division, dividing, divide, divided by, divided into, left, left over, remainder, grouping, sharing, share, share equally, one each, two each, three each ... ten each, group in pairs, threes ... tens, equal groups of, doubling, halving, array row, column number patterns, division fact, inverse

Appendix 1: Overview of the calculations and the year group in which they are introduced

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Solve problems through practical methods use of numberlines/100 squares etc.	Partition numbers to aid addition - start to arrange in columns	Introduce column addition with numbers up to 3 digits and numbers with 1 decimal place	Column method - 4 digits - and numbers with up to 2 decimal places	Column method more than 4 digits	Multi digit calculations
Subtraction	Solve problems through practical methods use of number lines/100 squares etc.	Partition to aid subtraction - use of numberlines and 100 squares	Introduce column subtraction (decomposition) up to 3 digits - and numbers with up to 2 decimal places	Column method 4 digits- and numbers with up to 2 decimal places	Column method more than 4 digits	Multi digit calculations
Multiplication	Solve problems through practical methods and knowledge of tables including using the signs x and = correctly	Use arrays / repeated addition / multiplication facts / practical resources including using the signs x and = correctly	Efficient written method -2 digit by 1 digit column method $\begin{array}{r} 38 \\ \times 7 \\ \hline 266 \\ \hline \end{array}$	Efficient methods 2 and 3 digit numbers by 1 digit number	4 digit numbers by 1 and 2 digits (long multiplication for 2 digits) Introduce long multiplication	Multiply multi-digit numbers up to 4 digits by a 2 digit whole number using long multiplication
Division	Solve problems through practical methods including using the signs = and ÷ correctly	Use arrays / repeated addition and subtraction / division facts / practical resources including using the signs = and ÷ correctly	Introduce the efficient method $4 \overline{) 48} \begin{array}{l} 12 \\ 48 \end{array}$ No remainders	Secure the efficient method - no remainders (Can include decimals)	Divide 4 digit by 1 digit numbers and interpret remainders appropriately for context (fractions or whole number remainders) Money as well	Divide up to 4 digits by a two digit whole number using long division

Appendix 2: Grading of Difficulty for questions

Below is a grid that lays out an order in which questions can be given to children looking at the difficulty of working out each operation.

Addition	Subtraction	Multiplication	Division
<p>1.No carrying e.g.</p> $\begin{array}{r} 23 \quad 315 \\ + 42 \quad + 624 \end{array}$	<p>1. No adjustment e.g.</p> $\begin{array}{r} 47 \quad 864 \\ - 23 \quad - 621 \end{array}$	<p>1. No carrying e.g.</p> $\begin{array}{r} 32 \quad 44 \\ \times 3 \quad \times 2 \end{array}$	<p>1. Single digit division, no remainder, no carrying e.g.</p> $69 \div 3 \quad 264 \div 2$
<p>2. Extra digit in answer e.g.</p> $\begin{array}{r} 94 \quad 561 \\ + 73 \quad + 718 \end{array}$	<p>2. Adjustment T to ones e.g.</p> $\begin{array}{r} 52 \quad 432 \\ - 36 \quad - 217 \end{array}$	<p>2. Extra digit in answer e.g.</p> $\begin{array}{r} 32 \quad 51 \\ \times 4 \quad \times 4 \end{array}$	<p>2. Remainder, no carrying e.g.</p> $68 \div 3$
<p>3. Carrying ones to T (tens) e.g.</p> $\begin{array}{r} 47 \quad 237 \\ + 25 \quad + 516 \end{array}$	<p>3. Adjustment H to T e.g.</p> $\begin{array}{r} 437 \quad 618 \\ - 182 \quad - 217 \end{array}$	<p>3. Carrying but keeping in same decade e.g.</p> $\begin{array}{r} 23 \\ \times 4 \end{array}$	<p>3. No remainder, carrying e.g.</p> $45 \div 3$
<p>4. Carrying T to H e.g.</p> $\begin{array}{r} 371 \quad 293 \\ + 485 \quad + 541 \end{array}$	<p>4. Adjustment H to T and T to ones</p> $\begin{array}{r} 432 \\ - 187 \end{array}$	<p>4. Carrying and going into next decade e.g.</p> $\begin{array}{r} 78 \quad 68 \\ \times 7 \quad \times 8 \end{array}$	<p>4. Remainder, carrying e.g.</p> $47 \div 3$
<p>5. Carrying ones to T and T to H e.g.</p> $\begin{array}{r} 376 \quad 295 \\ + 485 \quad + 547 \end{array}$	<p>5. Noughts e.g.</p> $\begin{array}{r} 470 \quad 700 \quad 604 \\ - 142 \quad - 236 \quad - 347 \end{array}$	<p>5. Noughts e.g.</p> $\begin{array}{r} 202 \quad 430 \\ \times 4 \quad \times 6 \end{array}$	<p>5. Placing of the quotient e.g.</p> $287 \div 7$
<p>6. More than two numbers to be added e.g.</p> $\begin{array}{r} 463 \\ 921 \\ + 759 \end{array}$		<p>6. Multiplying by multiples of 10</p> $\begin{array}{r} 87 \quad 416 \\ \times 10 \quad \times 60 \end{array}$	<p>6. Noughts in quotient e.g.</p> $816 \div 4 \quad 5608 \div 8$
<p>7. Different numbers of digits e.g.</p> $\begin{array}{r} 23 \quad 4756 \\ 375 \quad 20375 \\ + 48 \quad + 752 \end{array}$		<p>7. Long multiplication e.g.</p> $\begin{array}{r} 47 \quad 832 \\ \times 23 \quad \times 74 \end{array}$	<p>Two-digit division</p> <p>7. No remainder e.g.</p> $64 \div 32 \quad 93 \div 31$
			<p>8. Similar but remainder e.g.</p> $29 \div 13 \quad 97 \div 31$
			<p>9. Quotient not so apparent e.g.</p> $56 \div 22 \quad 92 \div 41$
			<p>10. Placing the quotient e.g.</p> $126 \div 21 \quad 224 \div 32$
			<p>11. No remainder e.g.</p> $483 \div 21 \quad 736 \div 32$
			<p>12. Remainder e.g.</p> $718 \div 33$
			<p>13. Noughts in quotient e.g.</p> $6834 \div 17$
			<p>14. Divisors like 29, 39, 48</p>
			<p>15. Divisors like 45, 37, 24, 56</p>

Appendix 3 – Bar Models (see separate document)

Appendix 4 Number Lines (see separate document)